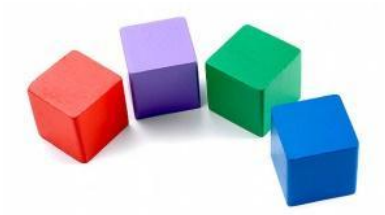
Name: _____Ch 3 Combinations – 3.1 MDM 4U1

3.1 – Permutations with Non-ordered elements



A. Investigation: Permutations with Repeated/Identical items

You are given four blocks: Two white blocks, a red block and a green block.

1. How many ways can you put the items in order, if you treat each of the two white blocks as **unique items**? (i.e. the white blocks are labeled W_1 and W_2)

2. If the two white blocks are now treated as **being identical,** how many distinct (i.e. truly different) ways can you put the items in order? Try physically moving the blocks and list all the distinct options to help you.

3. For each of your arrangements of WWRG (where the white blocks are treated as identical), **list all the matching/similar arrangements** that exist if the white blocks were treated as unique. An example is given.

Arrangement of WWRG (White blocks treated as identical)	Matching Arrangements of W_1W_2RG (White blocks treated as unique)
RGWW	RGW_1W_2 or RGW_2W_1

4. Notice that, when you have *some items that are identical,* the number of distinct (i.e. truly different) possibilities is **reduced.** Can you come up with an **expression**, using factorials, to show the number of distinct possibilities **of 4 items**, where **2 of them** are identical?

5. Can you come up with a **general expression**, using factorials, to show the number of distinct possibilities of ‘n’ items, where ‘a’ of them are identical?

--stop here and check in with your teacher before continuing----

Examine the next page **before** attempting the questions below.

B. Permutations with Repeated Items - Practice

1. How many different permutations are there of the letters in the word MOOSE?

2. In how many different ways can three identical copies of one novel and four other different novels be arranged on a shelf?

3. Tsering is going to randomly plant 10 rosebushes in a row. Six of the bushes are white, two are yellow and two are red. Find the number of distinct ways of planting the rosebushes.

4. “Are you not entertained?”

   a) How many (distinct) permutations are there of the letters of the word ENTERTAINMENT?

   b) How many of the arrangements begin with R?

   c) How many of the arrangements begin with T?

   d) In how many of the arrangements would the three T’s be together?

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Permutations with Identical Items

Notice in the first example, that there are 2! ways or organizing W1 and W2 in each arrangement of WWRG. If the white blocks are treated as identical, then the 2! ways to organize W1 and W2 no longer matter. Therefore, to find the number of distinct (truly different) permutations of WWRG (where the white blocks are no longer treated as being different), ***we need divide out 2!***

Suppose you want to know how many ways you can arrange a set of n items. If some number of these items, 'a', are identical, then you need to **undo or divide out** the $a!$ ways you would have arranged those items within the set if they were distinct. If you also had 'b' identical items, you would also divide out the $b!$ ways of organizing those items too.

The number of permutations of ***n*** objects, of which ***a*** are identical items of one kind, ***b*** are identical items of a second kind, ***c*** are identical items of a third kind, … and so on is:

$$Permutations = \frac{n!}{a!b!c!...}$$

---stop here and check in with your teacher before continuing----

C: "NUMB3RS" - Distinct objects in a fixed order.

How many ways are there to arrange the letters in the word NUMBER if the consonants must remain in their original order? (that is, N, M, B, R)

Hint! If the letters NMBR must all be in that order, in each arrangement, then you can pretend that all the consonants, NMBR, are identical items (which we'll call 'z').

In other words, NUMBER becomes zUzzEz . Use this to complete the question!

(Why does this work? Because, in each arrangement of 6 letters, there is only 1 way of organizing NMBR that matters, compared to all the ways you can organize the four letters, N,M,B,R. Hence you *divide out* all the ways you can organize the letters N, M, B, R.)

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Then try 3.1 HW pg 109 # 1- 8